## Investigation Chapter 1: Tire Story Falls Flat

PARTI
A legendary story on college campuses concerns two students who miss a chemistry exam because of excessive partying but blame their absence on a flat tire. The professor allows them to take a make-up exam, and sends them to separate rooms to take it. The first question, worth five points, is quite easy. The second question, worth ninety-five points, asks: Which tire was it?

## Step 1: Ask a research question

Do students pick which tire went flat in equal proportions? It has been conjectured that when students are asked this question and forced to give an answer (left front, left rear, right front, or right rear) off the top of their head, they tend to answer "right front" more than would be expected by random chance.

## Step 2: Design a study and collect data

To test this conjecture about the right front tire, a recent class of 28 students was asked if they were in this situation, which tire would they say had gone flat. We obtained the following results:

| Left front | Left rear | Right front | Right rear |
| :---: | :---: | :---: | :---: |
| 6 | 4 | 14 | 4 |

1. What are the observational units?
2. What is the variable that is measured/recorded on each observational unit?
3. Describe the parameter of interest in words. (You can use the symbol $\pi$ to represent this parameter.)
4. State the appropriate null and alternative hypotheses to be tested.

## Step 3: Explore the data

5. What percentage of the students picked the right front tire? Is this more than you would expect if students randomly pick one of the four tires?
6. Is it possible that we could observe 14 students from this class of 28 students even if all of the students were just selecting randomly among the four tires?

## Step 4: Draw inferences

Let's use our 3S strategy to help us investigate how much evidence the sample data provide to support our conjecture that there is something special about the right front tire.

## Statistic

7. What is the statistic that you can use to summarize the data collected in the study and what symbol is associated with this statistic?

## Simulate

8. Use the One Proportion applet to simulate 1000 repetitions of this study, assuming that every student in class selects randomly (equally) among the four tires. Report what values you input into the applet.

Probability of success $(\pi)$
Sample size ( $n$ ) $\qquad$
Number of samples $\qquad$
9. Using the proportion of successes for the values on the horizontal axis, what is the center of your null distribution? Does it make sense that this is the center? Explain.

## Strength of evidence

10. Let's examine the strength of evidence with the three ways we used in Chapter 1.
a) Determine the $p$-value from your simulation analysis. Also interpret what this $p$ value represents (i.e., the probability of what, assuming what?).
b) Determine the standardized statistic from your simulation analysis. Also interpret what this standardized statistic represents.
c) Use a theory-based test (or normal approximation) to determine the p-value for this test. Are the validity conditions met for this test? Why or why not?
11. Is there strong evidence against the null hypothesis for all three of the methods used in Question 11? Explain.

## Step 5: Formulate conclusions

12. Summarize the conclusion that you draw from this study and your simulation analysis. Also explain the reasoning process behind your conclusion.
13. Now, let's step back a bit and think about the scope of our inference. What are the wider implications? Do you think that your conclusion holds true for people in general? (These are extremely important questions, that we'll discuss more when we talk about the scope of inference in Chapter 3.)

## Step 6: Look back and ahead

14. If you were to repeat this study, what improvements might you make? What further research might you propose related to this topic in the future?

PART II
Now suppose another class conducts the same study with exactly half as many students, and suppose the proportional breakdown in the four categories is identical to the class of 28 . In other words, 7 out of 14 students answered, "right front."
15. Before you analyze the data, would you expect to find stronger evidence for the research conjecture (that people pick the right front tire more than $1 / 4$ of the time), weaker evidence, or the same strength of evidence? Explain your thinking.
16. Conduct a simulation analysis to produce a simulated $p$-value. How does it compare to the p-value from the study of the class with 28 students? Is this what you expected? Explain.

## PART III

Suppose we didn't have a preconceived notion that the right front tire would be chosen more often, but just wanted to find out if it was chosen at a rate that was different than one-fourth. Let's use our original data where 14 out of 28 chose the right front tire to test this.
17. Write the null and alternative hypotheses for this new question.
18. Use a theory-based test to find the p-value. How does this compare with the p-value you obtained back in Question 11c.
19. Do you have strong evidence that the probability a student will choose the right front tire is different than one-fourth?

